INDEX NUMBERS AND SCORING OF WATER SUPPLIES

By Abel Wolman

At the 1918 annual convention of the American Water Works Association there was presented a paper[1] by Mr. Robert B. Morse and the author on "The Practicability of Adopting Standards of Quality for Water Supplies." In that discussion an attempt was made to indicate the practical difficulties in the way of formulating a standard for water supply quality. As a member of the 1918 Committee on Standards for Water Supplies, the author has placed before himself the further task of discussing the principles of standardization or of its analogue, the scoring of water supplies. For even after the practical difficulties of bacteriological technique, of interpretation, and sanitary survey have been solved, the method and the logic of scoring still remain to be analyzed. In order to examine the principles of scoring, it has been necessary to formulate them with some exactness. In the search for such principles in other sciences, it was soon apparent that the choice and construction of index numbers in the field of economic statistics afford a problem so closely analogous to that of the standardization of the quality of water supplies that it was thought desirable to make use of some of the concepts already so carefully developed in the former field.

This paper has for its object, therefore, the presentation of the principles of water supply scoring, a comparison of these methods with similar ones in index number making, the discussion of their points of analogy and difference, and the development of certain data as guides in the more simple problems of water supply standardization. The first part of the paper will deal with the general aspects outlined above, while the second part will present the more specific material suggested in the above program.

[1] See Journal, September, 1918, p. 198.

SCORES AND INDEX NUMBERS

The quality of a water supply is a complex function of a series of attributes of varying importance. The measure of the potability of a supply, in the broad sense, is simply a weighted average, while the scoring of a water supply becomes, in like manner, a process of developing a weighted average, a single expression disclosing all the significant characteristics of the complex data which go to make up the concept of "potability." When we speak of the quality of water we mentally focus our judgment upon the various factors of bacterial and chemical analyses, of control of purification systems, and of sanitary surveys. By comparisons of different waters an unconscious weighting of attributes results. This necessity of weighting suggests the further search for quantitative, concentrated or summarizing expressions. The development of such expressions in the field of water supply quality has issued in the form of a standard or score. When this conception of the measure or standard of quality as a weighted average is extended to include comparisons of waters at different places and at different times, we approach the concept of the index number developed in statistical method. This analogy between the score and the index number is important, since the study of the latter points out the possible advantage in or the barriers to the use of the former.

Secrist states[2] that

> The purpose of an index number is to reduce to a common denominator the qualities of different factors or phenomena so as to allow comparison. . . . . It is to translate absolute into relative qualities in order that comparisons may be made. Moreover, index numbers are summaries direct or indirect of things having a common quality. . . . . They represent this quality as an aggregate or average at different times for purpose of comparison.

He explains further that such index numbers represent divergent things, responding differently to conditions and occupying different positions in the economy of business.

In a similar manner the problem with which the sanitarian is usually concerned is the comparison of different qualities of the same or different supplies, at the same or different times. He is

[2] Secrist, Horace, An Introduction to Statistical Methods, p. 297, Macmillan.

confronted, therefore, with difficulties similar to those of the economist who attempts to study the general drift of prices, of wages, or of rents. In each case a "direct or indirect summary of things" having a common, but not necessarily the same, significance is essential. The economist, through a number of years of study, has found the solution to his problem, and a survey of his method of developing the index number and its principles will point the way for the student of water supply scoring. It remains to be seen, however, whether the same principles may be adopted in both fields.

The operations involved in making a price index number, for instance, are similar to those followed, to a greater or less extent, by the various investigators of water supply scores. These operations, adapted to our problem of water supply scoring, are briefly summarized[3] as follows:

(a) Defining the purpose for which the final results are to be used.

(b) Deciding the numbers and kinds of conditions or concepts to be included.

(c) Determining whether these characteristics shall all be treated alike or whether they shall be weighted according to their relative importance.

(d) Collecting the actual figures for the characteristics chosen and, in case a weighted series is to be made, collecting also data regarding their relative importance.

(e) Deciding whether to measure the average variations of characteristics or the variations of a sum of actual characteristics.

(f) In case average variations are to be measured, choosing the base upon which relative qualities shall be computed.

(g) Settling upon the form of average to be struck.

A study of the above series of processes discloses the close analogy between the ideas underlying the development of an index number and those leading to the formation of a standard of quality. The factors outlined in (a), (b), and partially in (d) have been recognized generally in earlier attempts at standardization. With few exceptions, little has been attempted in the way of developing the ideas under (c), that is, determining whether the characteristics

[3] Adapted from Mitchell, W. C., Index Numbers of Wholesale Prices in the United States and Foreign Countries, *Bulletin of the U. S. Bureau of Labor Statistics*, Whole No. 173, July, 1915.

shall all be treated alike or whether they shall be weighted according to their relative importance.

Just as in price index numbers, "a change in the price of sulphuric acid, for example, means more to business and industry at large than the same change in the price of hydrochloric acid,"[4] so in water supply standards or scores, a change in B. coli content may have more significance than a concomitant change in the total count. The index number, therefore, in each case should be so arranged "that the first change would affect the final result more powerfully than the second."[4] The practical development of this principle is illustrated in making the index numbers for different American industries, where "the monthly price of each commodity included in the index number was multiplied by the quantity produced in and imported into the United States, the products of these multiplications were added up separately for each month in the six years covered, and the aggregates were turned into relatives on the prewar base. Under this plan great staples like bituminous coal, yellow-pine lumber, beef and cement, which are used each year in enormous quantities, exercise much more influence upon the final results than articles like horsehair, hickory, cinnamon and bone buttons, whose combined production and imports are small."[4]

It is apparent that a similar differentiation of influence of factors upon the index number in water supply quality is a desideratum. It might be helpful, of course, to be able to weight with such exactness, as indicated in the above example of price index numbers, the B. coli content of a tap water against the objectionable sanitary conditions upon a watershed and obtain a score or index number. Attempts at such weighting have been made by some investigators. The similarity of function of the index number and the score being granted, are we justified, then, in assuming that the processes of development of the former may be or can be applied to the formation of the latter? In other words, does not the analogy between water supply scoring and the economist's index number making end with the similarity of their functions? Are sanitarians justified in carrying forward the principles of weighting beyond the recognition of their necessity?

The answer to this query will be more apparent in the further consideration of the methodology of index number making.

[4] Mitchell, W. C., International Price Comparisons, War Industries Board, *Bulletin No. 2*, pp. 2-3.

In the case of price index numbers, the object of weighting is to give each commodity included in the index number an influence upon the results corresponding to its *commercial* importance. In water supply index numbers, the object of weighting likewise is to give each factor making up the score an influence upon the results corresponding to its *sanitary* importance. The problems in the two fields are the same, but their solutions are widely different. In fact a wide-spread system of weighting the attributes of water supply quality appears impossible of development. A further comparison of the necessary processes involved in the formation of the two different index numbers should make this conclusion clearer. In price index numbers the weighting is usually a simple process of assembling similar units into aggregates. The weighting rarely, if ever, demands the conversion of dissimilar units into a composite, but presupposes in most cases such convertible units as the values of pounds and tons, and dollars. No one of the steps is dependent upon the investigator's opinion of the value of a pound of any commodity in dollars, since this conversion factor is predetermined by economic forces and not by personal judgment. It becomes possible, therefore, in such a system of weighting, for investigator A to form the same index number from a series of commodities as investigator B, since both are provided with the general equations converting the values of barrels, tons, and gallons, into dollars. The resulting figure for the index number depends, therefore, upon the choice of factors and not upon the individual's personal estimate of their relative importance.

In water supply scoring, however, the choice of factors, bacterial results, sanitary survey, operation, etc., are fairly well agreed upon, but the consideration of their relative importance remains the barrier to the successful application of index number making. This barrier seems to the author to be a permanent one, certainly as far as the national adoption of water supply index numbers is concerned, since the necessary conversion factors for the various units in a complex water supply standard are lacking. It is entirely doubtful, for instance, whether it is possible or desirable to attempt to combine the B. coli content of a tap water with the degree of excellence of filter plant operation in order to obtain an inclusive quantitative measure. The truth of this statement is apparent, when it is borne in mind that it is not logically permissible to combine incommensurables and to obtain in this way a real index number.

The author believes that any attempts to score water supplies upon a national scale by the weighting of their different sanitary characteristics fail to observe the precept that the conversion of units to a common base presupposes the absence of varying personal opinion or judgment, and the converse proposition that factors subject to differences of personal equations are incommensurable.

The above fundamental barrier, however, would appear to prevent only the use of a national standard of *complex form.* It still remains possible to make use nationally of *simplified* index numbers restricted in their range of significance and composed of similar units or, better still, of only individual units, provided the method of evaluation of such units has been definitely and completely fixed.

The latter provision has not yet been met for the country at large, but is or may be accomplished in a number of individual states, where water supplies are state controlled and, hence, where units of measurement may be made standard.

Starting, therefore, with the existence of similar methods of unit evaluation within a state, it is possible by the accumulation of basic data to institute a system of restricted or simple index numbers or measures of variation. Even in such intra-state systems, however, only comparable units can be studied, while structural, environmental, and operative features have their effect in the consideration of quality only through the qualitative judgment and experience of the student, rather than by the substitution, therefore, of a pseudo-quantitative index of doubtful significance.

In order to assist the investigator in his problem, experience data become necessary. The desirability of continued concerted study of pertinent water supply data is patent, therefore, in view of the fact that it is only by such study that basic data are accumulated to supply a knowledge of the conditions of normality. In order to illustrate such a procedure, the author has devoted the rest of this paper to the development of such data as the above. The data here reported are to serve as a guide for future studies of a *filtered chlorinated tap water in Maryland.* These restrictive limitations of the guide are used advisedly, since it may be found to vary for different classes of water supplies in the same state or for the same kind of water supply in different states. The present discussion will be restricted to only a portion of one phase of the process of simplified index number making, namely the collection, tabulation, and interpretation of the figures for a few of the characteristics of

the tap water described. An endeavor will be made to point out the significance of such a study of mass phenomena and to obtain therefrom some conclusions relative to the factors which influence index numbers or standards of quality of great simplicity.

The material on filtered chlorinated tap water here presented is an example of data which constitute one element of an index number and is material of such character that comparative data for the country as a whole may and can be ultimately obtained. In the same way, similar groups of comparative material on other simple attributes may be collected and thus will provide the bases for other comparison.

DATA FOR A SIMPLIFIED INDEX NUMBER

1. Method of tabulation and calculation. As the initial step in this study of index numbers of quality, it was determined to make use of the 1916, 1917, and 1918 figures for the 20°C. and 37°C. bacterial contents[5] of a tap water in a city in Maryland. This city was chosen because the frequency of sampling was sufficiently great to eliminate chance errors of testing, because the city has a surface water supply similar to that of a large number of American cities, which is in addition subjected to the processes of purification, rapid sand filtration and chlorination common to other water supplies. The tap waters used may be considered, therefore, in their characteristics, if not in their absolute values, typical of American cities. All of the samples to be considered in this discussion are tap samples collected daily throughout the year in various parts of the city. The average number of samples used in each temperature series is approximately 1500 in each year. Thus the total number of samples tabulated aggregated about 3000 per year, or an approximate total for the three years of 9000 samples.

In order to grasp the significance of such a record of thousands of values of variables, it was necessary to classify the observations in each series. This was done by monthly and annual series. The latter only are reported here (table 1), while the former have been omitted to conserve space. The statistical rule that class intervals should be all equal was broken, principally in order to condense an otherwise unwieldly table. The result of such a change is not

[5] 37°C, 24 hours, agar; 20°C, 48 hours, agar.

significant, when it is borne in mind that the rule has been fairly closely followed in those classes in which the major portion of the observations fall.

2. Analysis of Observations. Yule[6] describes the manner in which the observations are distributed over the successive intervals of the scale as the frequency-distribution of the variable. Too often in the past, the standards for water supplies have neglected almost

TABLE 1

Variation of bacterial content of tap water

TOTAL SAMPLES	YEAR	37°C. BACTERIA PER CUBIC CENTIMETER BETWEEN									
		0–25	26–50	51–75	76–100	101–150	151–200	201–300	301–400	401–500	501 and over
1284	1918	1036	145	28	17	14	10	13	9	1	11
		81	92	94	95.0	97.0	97.5	98.5	99.0	99.0	100.0
1562	1917	1186	159	45	44	37	29	32	13	3	14
		76	86	89	92.0	94.0	96.0	98.0	98.5	99.0	100.0
1526	1916	994	313	74	55	32	12	15	9	3	17
		65	85	89	94.0	96.0	97.0	98.0	98.5	98.8	100.0
		20°C. BACTERIA PER CUBIC CENTIMETER BETWEEN									
		0–25	26–50	51–75	76–100	101–150	151–200	201–300	301–400	401–500–	501 and over
1447	1918	998	156	48	34	31	20	34	29	9	88
		69	80	83	86.5	87.5	89.0	91.0	93.0	93.0	100.0
1541	1917	1007	136	34	49	35	33	24	24	10	191
		65	74	76	80.0	82.0	84.0	85.0	87.0	87.5	100.0
1526	1916	775	248	82	71	67	24	42	22	20	175
		51	67	72	77.0	81.5	83.0	85.5	87.0	88.5	100.0

Note.—Upper figures represent number of samples in each class, lower figures the cumulative percentages of the total number of samples.

completely the factor of frequency-distribution. Most of the standards have been concerned with averages of various kinds, neglecting the primary fact that in sanitary importance the deviations from the normal in water supplies have been almost always of more significance than the absolute normal. Typhoid fever epidemics have had their origin, in the water-borne instances, in the abnormal

[6] Yule, G. U., An Introduction to the Theory of Statistics, p. 77, Griffen & Co., Ltd.

conditions of water supplies and not in their usual. The frequency-distribution of results visualizes these important variations. For convenience of representation, the graphical presentation of the frequency-distribution is here used in addition to the tables of figures.

In order to determine the nature of the distribution of the different bacterial results in the various years studied, the cumulative values shown in the tables have been plotted in figures 1, 2, and 3. The values are plotted on the so-called probability paper developed by Allen Hazen in 1913. This paper has been ruled with lines spaced in accordance with a probability curve or according to the normal law of error. The method used in the construction of the lined sheets will not be discussed here, since it is fully described in the original contribution by Hazen.[7] It is important to note, however, that "if the data for any series correspond strictly with the normal law of error, the points plotted on this paper will all be in a straight line." An inspection of figures 1, 2, and 3 indicates at once that the tap water results for the three years in the city under discussion coincide fairly well with the normal law of error. The nature of the distribution of these results, namely the logarithmic-probability type, is entirely similar to that reported by Prof. G. C. Whipple for Philadelphia and Springfield filtered waters.[8]

3. Significance of findings. The collection and representation of a large number of tap water results as herein used make it possible to deduce certain general conclusions regarding the future study of the quality of potable waters by such regulating bodies as state departments of health. The charts shown in this paper indicate, for instance, the practical conclusion that little is lost in critical studies of water supply by using either one of the two bacterial criteria, the 20°C. or the 37°C. counts, in place of the other. The conclusion is attested by the fact that each of these has a similar mass distribution, differing one from another, only in amount and not in nature. If the investigator takes sufficient pains to determine the proper historical base for each of the indices, *by a study of past results*, it is immaterial for future control which count he uses. It is remembered, of course, that different bases are used

[7] Hazen, Allen, Storage to be Provided in Impounding Reservoirs, *Trans. A. S. C. E.*, lxxvii, p. 1539.

[8] Whipple, G. C., Element of Chance in Sanitation, *Jour. Franklin Inst.*, vol. 182, no. 1, July, 1916.

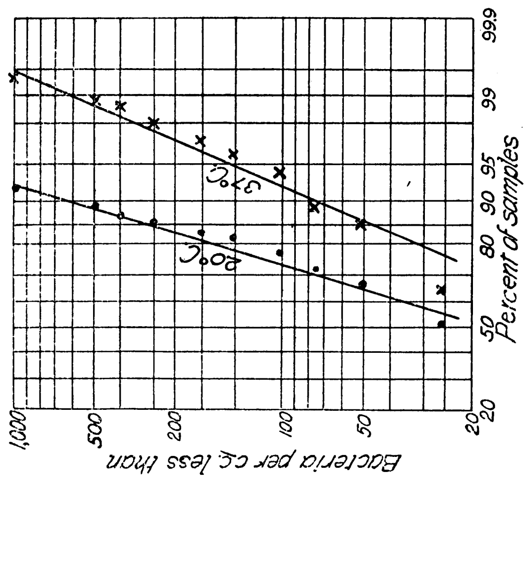

Fig. 1, 1916

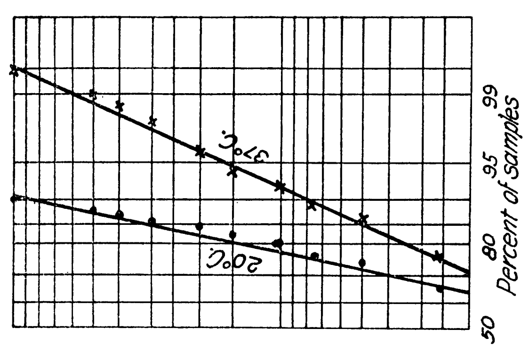

Fig. 2, 1917

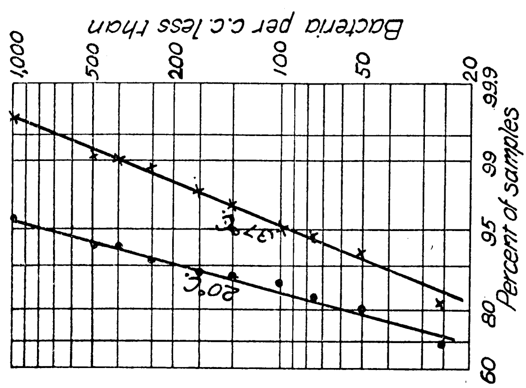

Fig. 3, 1918

Frequency-Distribution Records of Bacteria in Filtered and Chlorinated Water

TABLE 2

Variation of bacterial content of tap water. Corrected values for use in standard quality sheet

TOTAL SAMPLES	YEAR	37°C. BACTERIA PER CUBIC CENTIMETER BETWEEN								
		0–5	6–10	11–15	16–20	21–25	26–50	51–75	76–100	101–150
952	1918	372	237	97	54	30	89	16	13	11
		39	64	74	80	83	92	94.0	95	97
						(0-25)				
1449	1917					1164	151	35	26	19
						80	90	93.0	95	96+
		20°C. BACTERIA PER CUBIC CENTIMETER BETWEEN								
		0–5	6–10	11–15	16–20	21–25	26–50	51–75	76–100	101–150
1050	1918	405	185	102	73	33	109	24	21	11
		39	56	66	73	76	86	89.0	91	92
						(0-25)				
1272	1917					925	125	33	37	25
						73	83	85.5	88	90

Note.—Upper figures represent number of samples in each class, lower figures the cumulative percentages of the total number of samples.

TABLE 3

Variation of bacterial content in raw water. 1917 and 1918 corrected cumulative values and percentages

TOTAL SAMPLES	37°C. BACTERIA PER CUBIC CENTIMETER BETWEEN								
	0–500	501–1000	1001–5000	5001–10000	10001–15000	15001–20000	20001–30000	30001–40000	40001–50000
508	0	348	479	497	500	503	504	505	506
	0	70.0	95.8	99.4					
	20°C. BACTERIA PER CUBIC CENTIMETER BETWEEN								
	0–500	501–1000	1001–5000	5001–10000	10001–15000	15001–20000	20001–30000	30001–40000	40001–50000
509	0	104	331	371	394	412	432	443	457
	0	20.8	66.2	74.2	78.8	82.4	86.4	88.6	91.4

Note.—Upper figures represent number of samples in each class, lower figures the cumulative percentages of the total number of samples.

for comparison and, with these established, it becomes relatively easy to measure changes of quality in extended series by either one of the significant counts. The graphical procedure makes it possible also to check the findings in the one series of results with the corresponding findings at the other temperature. Here, again, the precaution of obtaining the proper preceding series for comparison must be observed.

In order to obtain a future guide for the study of the quality of a filtered chlorinated water, it was necessary to correct the values

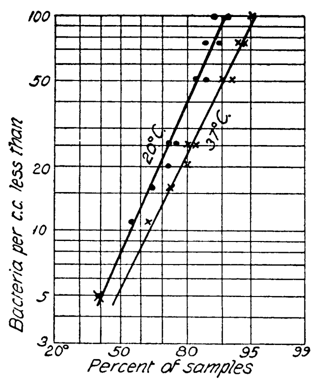

FIG. 4. ALLOWABLE FREQUENCY-DISTRIBUTION OF BACTERIA IN FILTERED AND CHLORINATED MARYLAND WATER

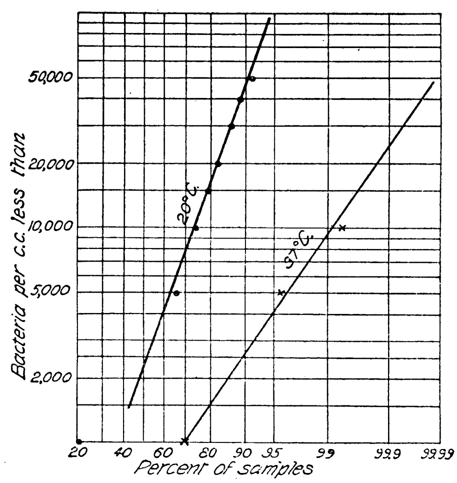

FIG. 5. FREQUENCY-DISTRIBUTION OF BACTERIA IN RAW WATER DURING 1917 AND 1918

shown in table 1, so as to eliminate bacterial results definitely established as due to inavoidable faulty conditions of operation or to unexpected temporary, but necessary, changes in source of supply. For the purpose of developing a base for comparison of future 20°C. and 37°C. counts, it was decided to eliminate the results obtained during 1916 and several months in 1917 and 1918. This was done to permit the use of bacterial results in a tap water during periods of normal operation, but excluding "tuning up" of purification processes, intervals of chemical shortages, and temporary changes in sources of supply subject to inadequate purification.

The revised list of data for the development of a filtered chlorinated water guide-post to be used in Maryland, is shown in table 2 and in figure 4, which serves as the "standard quality sheet," to be used for basic long period studies of the supplies similarly treated.

An inspection of the "standard quality sheet" for a filtered chlorinated water in Maryland indicates, for instance, that the median bacterial count in the tap water is about 6 for the 37°C. count and about 8 for the 20°C., while 95 per cent of the tap waters, in the series studied, show 37°C. counts less than 100 and 90 per cent indicate 20°C. counts less than 100 per cc. It is of interest to recall that Francis F. Longley in 1916 was reported to have found that "in well ordered filter plants the median bacterial count for the filtered water is usually less than 20 per cubic centimeter, while during 95 per cent of the time the bacterial counts are less than 100 per cubic centimeter."[8] It would be desirable, for purposes of judgment in control of various water supplies, particularly those under state supervision, to have more complete statistical data like the above from various parts of the country. For purposes of comparison, the bacterial load carried by the plant from which the above data were collected is shown on figure 5 and in table 3.